

Math 153 Pre Calculus I Cornerstone Final 2019-2020

Teacher/School \_\_\_\_\_ Class Time \_\_\_\_\_ Name \_\_\_\_\_

**Instructions.** Answers with no work may be counted incorrect. The grader must be able to understand your thought process from your work, so be clear. You may use your calculator on this exam. Write your answer in the box provided. **When an exact answer is not required, and no special rounding rules are indicated, round to 3 decimal places.**

1. Consider the function  $f(x) = x^5 - 5x^3 + 4x$ .

a. (4 points) Algebraically calculate the coordinates and the multiplicity of any x-intercepts for this function.

Work:

Answer

b. (4 points) Using your graphing calculator, find the x and y coordinates of all turning points. Use an appropriate x and y-window. Indicate whether each point is a local (relative) maximum or a local (relative) minimum.

Work:

Answer

c. (2 points) Using interval notation, list the interval (or intervals) on which the function is decreasing.

Answer

2. (4 points) Solve the equation algebraically. Give the exact answer. Simplify fractions and radicals when possible.  $x^2 + 4x + 1 = 10$

Work:

Answer

3. (4 points) **ALGEBRAICALLY** solve the following inequality, giving exact answers. Write your answer in interval notation.  $x^2 + 2x + 3 < 2x^2 + 4x$

Work:

Answer

4. If  $g(x) = \frac{x^2 - 6x + 8}{x^3 - 4x}$ ,

- a. (4 points) Solve for x algebraically if  $g(x)=0$ .

Work:

Answer

- b. (2 points) Show your work algebraically to determine the domain for  $g(x)$ . Show your answer in interval notation. (2 points extra credit for stating the range of  $g(x)$ .)

Work:

Answer

5. (4 points). For the function  $f(x) = -x^2 - 4x + 4$ , find the difference quotient in the form  $\frac{f(x+h)-f(x)}{h}$ .

Answer

6. A college football stadium seats 70,000 people. At a price of \$25 per ticket, the college usually sells 68,000 tickets. They are considering raising the price. Market experts tell them that for each \$2 increase in price, the college will sell 1000 less tickets per game.

- a. (4 points) If prices may increase by any dollar amount, find the ticket price that will maximize the revenue. Round to the nearest cent.

Answer

- b. (2 points Extra Credit) If the college also knows that each person attending a game spends an average of \$15 on concessions, explain how this would affect the formula used in part a.

Answer

7. Given  $f(x) = \sqrt{x-1}$ , and  $g(x) = \frac{1}{x^2+1}$ , determine each of the following, *simplifying* if possible, and *rationalizing*, if necessary.

a. (4 points) Simplify and state the domain of:  $\left(\frac{f}{g}\right)(x)$ .

*Answer*

b. (2 points) Simplify  $(g \circ f)(x)$ , the composite function of  $g$  composed with  $f$ . *Rationalize* if necessary.

*Answer*

c. (4 points) Find, simplify, and state the domain of  $f^{-1}(x)$  (the inverse of  $f(x)$ ). *Rationalize* if necessary.

NOTE: *The domain of  $f(x)$  may have to be limited to make the inverse a function.*

*Answer*

8. Consider the parent function  $f(x) = e^x$  and the transformed function  $g(x) = -\frac{1}{2}e^{3x} + 4$ , quantitatively explain the transformations to the parent function,  $f(x)$  that will produce  $g(x)$ .

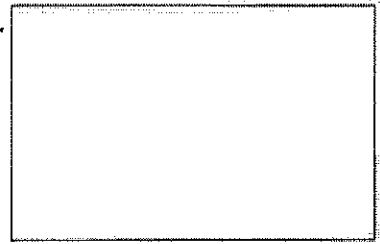
(4 points) *Answer*

9. Consider the function  $f(x) = \frac{2x^2 + 7x - 4}{2x^2 - 7x - 4}$ . Determine each of the following **ALGEBRAICALLY**.

a. (4 points) List the coordinates of x and y intercepts, if they exist. (Indicate if they do not exist.)

Work:

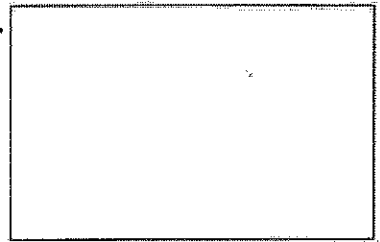
Answer



b. (4 points) Give the equations for any horizontal and/or vertical asymptotes (Indicate if they do not exist). Also, list the coordinates of any holes in the graph (Indicate if there are no holes).

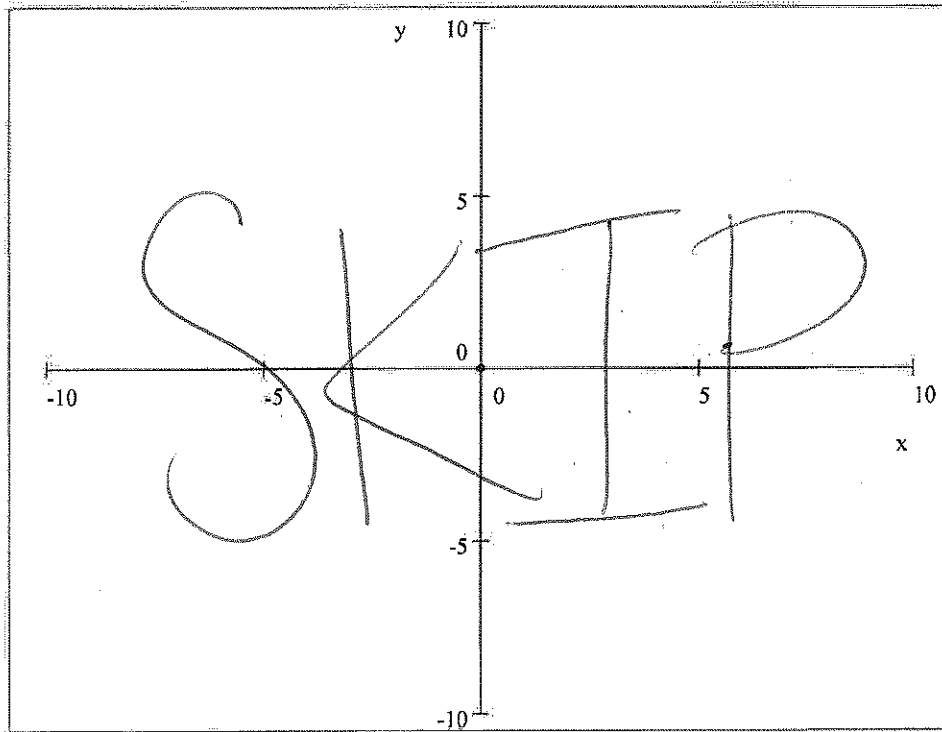
Work:

Answer



c. (4 points) Sketch the graph of  $f(x)$  in the space below. *Label all the features found in parts a and b.*

skip!



10. (4 points) Solve the following algebraically. Give exact answers.  $\ln(2x - 1) + \ln(3x - 2) = \ln 7$

a. Work:

*Answer*

11. (2 points) Simplify the expression  $e^{3 \ln 4}$ .

*Answer*

12. Solve both questions algebraically, **showing all work**. Give an exact answer for part *a* and use your calculator to find the decimal approximation for part *b* rounded to three decimal places.

a. (4 points)  $\frac{1}{5} \log_2(x - 1)^3 = 3$  only on the interval  $(1, \infty)$

*Answer*

b. (4 points)  $e^x = 2^{x+1}$

*Answer*

13. On a hot summer day in Death Valley, the amount of water in a swimming pool is 4000 gallons. 30 days later, the amount of water in the pool is 3500 gallons. If the evaporation follows a continuous exponential model for decay,

a. (4 points) Find  $k$ , the constant of proportionality in the equation  $A = A_0e^{kt}$ . Round to 4 decimal places.

Work:

*Answer*

b. (2 points) Write a function that models the amount of water in the pool  $A(t)$ ,  $t$  days after the original measurement.

Work:

*Answer*

c. (2 points) If this model continues to be applicable, find the time in which the amount of water would reach 2000 gallons. (Round to the nearest day.)

*Answer*