Final Answer Key (Practice Test)

1. $y = x^3 + 1$ over $[1,4]$ avg value

$$\frac{1}{b-a}\int f(x)\,dx$$

$$\frac{1}{4-1}\int_1^4 x^3+1\,dx$$

$$\frac{1}{3}\left[\frac{x^4}{4}+x\Big|_1^4\right]$$

$$\frac{1}{3}\left[68 - 1.25\right]$$

$$\boxed{= 22.25}$$

2. Let $f(x) = \ln x + e^{\sin x}$

Find $f'(1.602)$

$$f'(x) = \frac{1}{x} + \cos x\, e^{\sin x}$$

$$f'(1.602) = \frac{1}{1.602} + \cos 1.602\, e^{\sin 1.602}$$

$$\boxed{f'(1.602) = .539}$$

* Note: Can also do

nDeriv$(\ln x + e^{\sin x}, x, 1.602)$

3. Find max/mins for

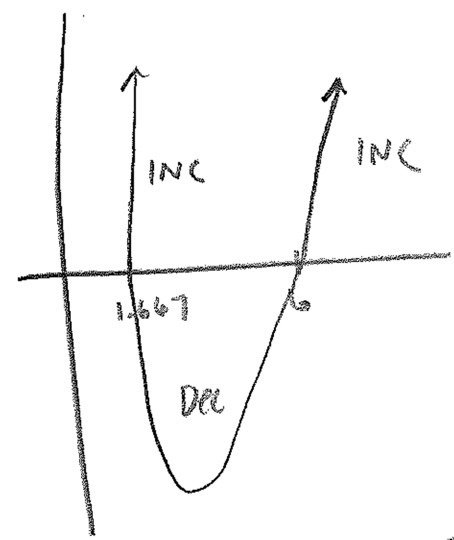
$V(x) = 4x^3 - 46x^2 + 120x$

3. cont...

$V'(x) = 12x^2 - 92x + 120$

↗ Graph and find zero's

$V'(x)$ graphed



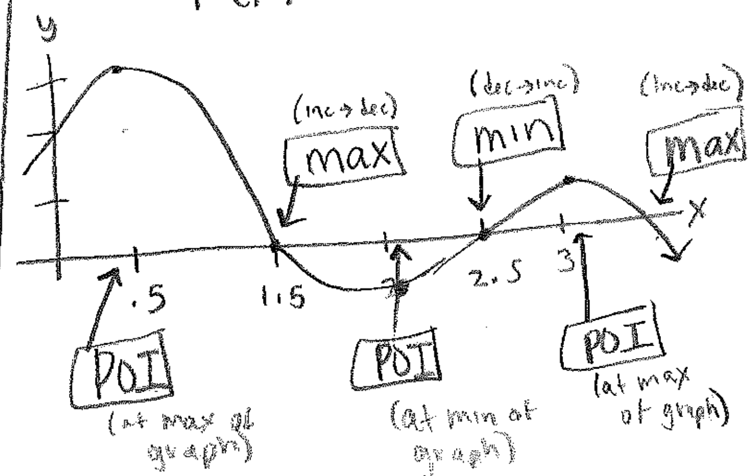
Inc → Dec @ $x = 1.667$

$\boxed{\text{Local max @ } x = 1.667}$

Decr → Inc @ $x = 6$

$\boxed{\text{local min at } x = 6}$

4. $f'(x)$



4. (the 2nd one woops)

① $F(x) = \int_2^{x^3} \frac{1}{t^3}\,dt$

$F'(x) = \frac{1}{(x^3)^3}\cdot(3x^2)$

$= \frac{3x^2}{x^9} = \boxed{\frac{3}{x^7}}$

② $F(x) = \int_x^{2x} \frac{2}{t}\,dt$

$\int_x^{c} \frac{2}{t}\,dt + \int_c^{2x} \frac{2}{t}\,dt$

$-\int_c^{x} \frac{2}{t}\,dt + \int_c^{2x} \frac{2}{t}\,dt$

$F'(x) = \frac{-2}{x} + \frac{2}{2x}\cdot 2$

$\frac{-2}{x} + \frac{4}{2x} = \frac{-2}{x} + \frac{2}{x} \boxed{= 0}$

③ $F(x) = \int_x^{x^2} (t^2 - 8t + 11)\,dt$

$-\int_c^{x} (t^2 - 8t + 11)\,dt + \int_c^{x^2} (t^2 - 8t + 11)\,dt$

$F'(x) = -x^2 + 8x - 11 + ((x^2)^2 - 8(x^2) + 11)\,2x$

$F'(x) = -x^2 + 8x - 11 + (x^4 - 8x^2 + 11)\,2x$

$= -x^2 + 8x - 11 + 2x^5 - 16x^3 + 22x$

$\boxed{= 2x^5 - 16x^3 - x^2 + 30x - 11}$

5.

$\frac{dl}{dt} = -3 \quad \frac{dw}{dt} = 5$

$L = 12 \quad w = 5$

Find a) $\frac{dA}{dt}$ and b) $\frac{dP}{dt}$

a) $A = lw$

$\frac{dA}{dt} = \frac{dl}{dt}(w) + \frac{dw}{dt}(l)$

$\frac{dA}{dt} = -3(5) + 5(12) = -15 + 60$

$\boxed{\text{a) } \frac{dA}{dt} = 45 \text{ cm}^2/\text{sec}}$

b) $P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(-3) + 2(5) = -6 + 10$

$\boxed{\text{b) } \frac{dP}{dt} = 4 \text{ cm/sec}}$

6. Area btwn curves. First, let's graph!

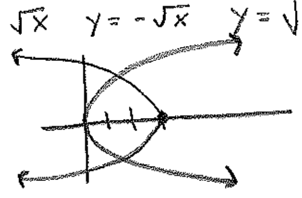
$x - y^2 = 0 \Rightarrow y^2 = x \quad y = \pm\sqrt{x}$

$x + 2y^2 = 3 \Rightarrow 2y^2 = 3 - x \Rightarrow$

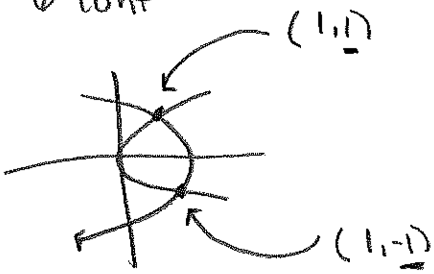
$y^2 = \frac{3-x}{2} \quad y = \pm\sqrt{\frac{3-x}{2}}$

* Note: You need to graph 4 functions

$y = \sqrt{x} \quad y = -\sqrt{x} \quad y = \sqrt{\frac{3-x}{2}}, \quad y = -\sqrt{\frac{3-x}{2}}$



6 cont



Must put integral interms of y and bounds in terms of y and do Right curve - left

In terms of y

$$\int_{-1}^{1} (3-2y^2)-(y^2)\, dy$$

Right Left

Right curve: $x+2y^2=3$

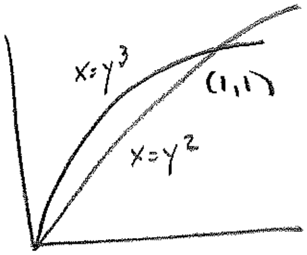
$x=3-2y^2$

Left: $x-y^2=0$, $x=y^2$

$$\boxed{\int_{-1}^{1} (3-2y^2)-y^2\, dy = 4}$$

(Just do this in your calculator with x)

7.



Right - Left Horizontal Method

In terms of y

$$\int_0^1 y^2-y^3\, dy \; \boxed{=.083}$$

Right - Left

7 cont... In terms of x (top - bottom)

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Solve for y

$y=\sqrt[3]{x}$ and $y=\pm\sqrt{x}$

But limit to (+) given picture

In terms of x

$$\int_0^1 x^{1/3}-\sqrt{x}\, dx \; \boxed{=.083}$$

top bottom

8.

1) $h_1'(2) = f'(2)+g'(2) = -1+3/2 = \boxed{1/2}$

2) $h_2'(3) = f'(3)-g'(3) = -1-1 = \boxed{-2}$

3) $h_3'(4) = f'(4)g(4)+g'(4)f(4) =$

$(-1)(5)+(1)(2)$

$-5+2 \; \boxed{-3}$

4) $h_4'(2) = \dfrac{f'(2)g(2)-g'(2)f(2)}{(g(2))^2} = \dfrac{-9}{(3)^2}$

$= \dfrac{-9}{9} = \boxed{-1}$

5) $h_5'(x) = 2f(x)\cdot f'(x)$

$2f(2) * f'(2) =$

$8 * -1 = \boxed{-8}$

6) $h_6'(6) = f'(g(6))g'(6) =$

$f'(4)\, -2 =$

$(-1)(-2)$

$\boxed{2}$

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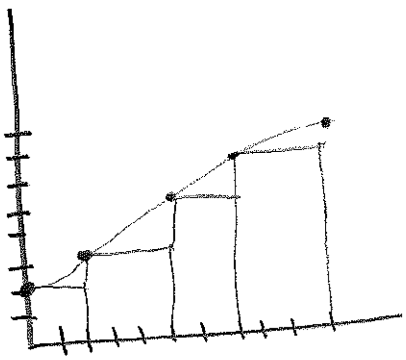
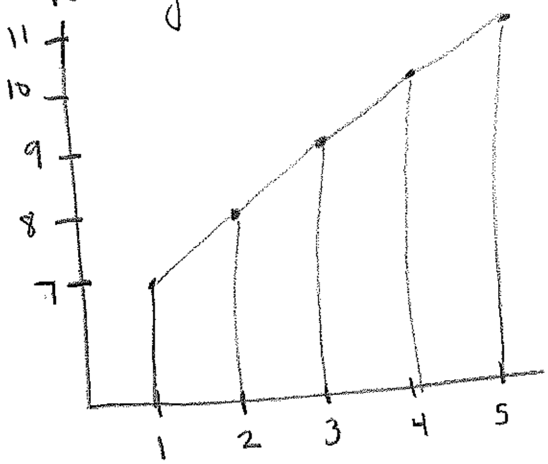
9. Estimate the integral w/ LRAM



$$\int_0^{10} f(x)\,dx \approx 2(2) + 3(3) + 2(5) + 3(7) = 4+9+10+21 = \boxed{44}$$

10. $y = x+6$ $[1,5]$ trapezoidal method (4 sub-int.)

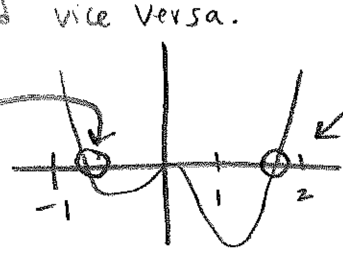


$$\int_1^5 (x+6)\,dx = \tfrac{1}{2}1(7+8) + \tfrac{1}{2}1(8+9) + \tfrac{1}{2}1(9+10) + \tfrac{1}{2}1(10+11) = \tfrac{1}{2}(7+8+8+9+9+10+10+11)$$

$$\boxed{= 36}$$

#11 This is the

11) graph of the 2nd deriv so look where the graph goes from + (concave up) to − (concave down) and vice versa.



POI

concave up to concave down

concave down to concave up

12. Recognize as derivative!

1) $f(x) = x^2$ $\boxed{f'(x) = 2x}$

2) $f(x) = \tan x$ $\boxed{f'(x) = \sec^2 x}$

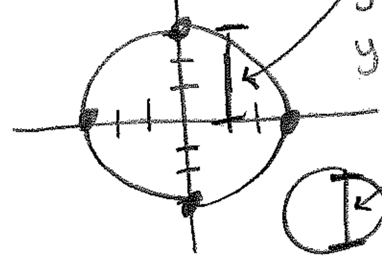
3) $f(x) = x^4$ $\boxed{f'(x) = 4x^3}$

4) $f(x) = 2x^2 - 3x$ $\boxed{f'(x) = 4x - 3}$

5) $f(x) = \sin(2x)$ $\boxed{f'(x) = 2\cos(2x)}$

6) $f(x) = \cos(x^2)$ $\boxed{f'(x) = -2x\sin(x^2)}$

13.



y distance

$y = \sqrt{9-x^2}$

$2\sqrt{9-x^2}$

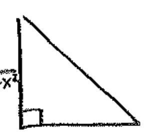
a) Square cross section $\left(\int A(x)\,dx\right)$

In terms of x

$$\int_{-3}^{3} \left(2\sqrt{9-x^2}\right)^2 dx = \int_{-3}^{3} 4(9-x^2)\,dx$$

$$= \boxed{144 \text{ units}^2}$$

b)



$2\sqrt{9-x^2}$

$2\sqrt{9-x^2}$

base height

$$A = \frac{1}{\cancel{2}}\left(\cancel{2}\sqrt{9-x^2}\right)\left(2\sqrt{9-x^2}\right)$$

$$A = 2\left(\sqrt{9-x^2}\right)\left(\sqrt{9-x^2}\right)$$

$$\int_{-3}^{3} 2(9-x^2)\,dx = \boxed{72 \text{ units}^2}$$

c)

$2\sqrt{9-x^2}$

$d = 2\sqrt{9-x^2}$

$r = \sqrt{9-x^2}$

$$A_{\text{semi-circle}} = \frac{\pi r^2}{2} = \frac{\pi\left(\sqrt{9-x^2}\right)^2}{2}$$

$$\frac{\pi}{2}\int_{-3}^{3} (9-x^2)\,dx = 36\pi/2$$

$$\boxed{= 18\pi \text{ units}^2}$$

14. $f(x) = 11x^2 - \frac{1}{x^2}$ @ $x=3$

$f(3) = 99 - 1/9$

We have the point $(3, 99.889)$

$f'(x) = 22x + 2x^{-3}$

$= 22x + \frac{2}{x^3}$

$f'(3) = 66 + \frac{2}{27} = 66.074 = m$

$\boxed{y - 99.889 = 66.074(x-3)}$

$y - 99.889 = 66.074x - 198.222$ OR

$\boxed{y = 66.074x - 98.333}$

15. $\pi \int R^2 - r^2 \, dx$

$\pi \int_0^2 (2x+2)^2 - (x^2+2)^2 \, dx = \frac{48\pi}{5} = \boxed{30.159}$

16. $\pi \int R^2$ (no inner radius)

$\pi \int_{-1}^{1} (-x^2+1)^2 \, dx = \frac{16}{15}\pi \approx \boxed{3.351}$

17. Outer Radius $-y^2+2 - -2 = -y^2+4$

Inner Radius: $y - -2 = y+2$

$\pi \int_{-2}^{1} (-y^2+4)^2 - (y+2)^2 \, dy$

$= \frac{108}{5}\pi = \boxed{67.858}$

18. $\frac{dy}{dx} = xy^2$ @ Point $(2, -2/5)$

$\int y^{-2} \, dy = \int x \, dx$

$-y^{-1} = \frac{x^2}{2} + c$

$\frac{-1}{y} = \frac{x^2}{2} + c$

$\frac{-1}{-2/5} = 2 + c$

$5/2 = 2 + c$

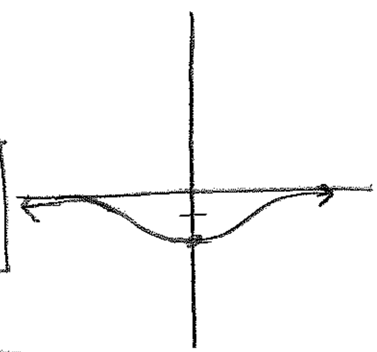
$5/2 - 4/2 = c, \quad c = 1/2$

$\frac{-1}{y} = \frac{x^2}{2} + \frac{1}{2}$

$\frac{-1}{y} = \frac{x^2+1}{2}$ Flip both Fractions!

$-y = \frac{2}{x^2+1}$

$\boxed{y = -\frac{2}{x^2+1}}$



18. $L(x) = f(a) + f'(a)(x-a)$

$f(x) = x^3$ $\underline{x = 2.02}$ $\underline{a = 2}$ $f(a) = 8$

$f'(x) = 3x^2$ $f'(2) = 3(2)^2 = 12$

$L(2.02) = 8 + 12(2.02 - 2) =$

$8 + 12(.02) = \boxed{8.24}$

19. $f(x) = \sqrt{x}$ $x = 8.9$ $a = 9$ $f(a) = 3$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

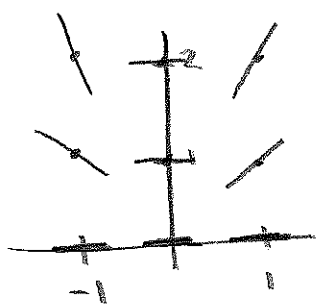
19 continued...

$L(8.9) = 3 + \frac{1}{6}(8.9 - 9)$

$\boxed{L(8.9) = 2.983}$

20. $\frac{dy}{dx} = xy^2$

$(-1,0)$	0	$(0,0)$	0	$(1,0)$	0
$(-1,1)$	-1	$(0,1)$	0	$(1,1)$	1
$(-1,2)$	-4	$(0,2)$	0	$(1,2)$	4



21. $V(t) = t\ln t - t$ $\quad P(1) = 6$ (Position)

a) $v'(t) = a(t) = \ln t + \overset{\frac{t}{t}}{1} - 1 = \boxed{\ln t}$

b) Find where $v(t) > 0$ ... (moving right)

$V(t) = t\ln t - t$

$t(\ln t - 1)$

$t > 0, \quad \ln t - 1 > 0$

$e^{\ln t} > e^{1}$

$\boxed{t > e}$

The particle moves to the right when $t > e$ because $v(t) > 0$ at $t > e$

c) To find minimum velocity, set $v'(t) = 0$

$e^{v'(t) = \ln t} = e^{0}$

$t = 1$

$v'(t)$: $-$ | $+$ at 1

$\boxed{\text{The minimum velocity is at } x = 1}$ because $v'(t) < 0$ for $t < 1$ and $v'(t) > 0$ for $t > 1$.

22.

a) $a(t) = 12t^2 - 4$

$V(t) = \int 12t^2 - 4$

$V(t) = 4t^3 - 4t + C$

at $t = 0$ $V(t) = 0$

$0 = 4t^3 - 4t + c$

$0 = 0 + c$

$c = 0$

$V(t) = 4t^3 - 4t = 0$

$4t(t^2 - 1)$

$\boxed{t = 0,\ t = 1}$

Not $t = -1$ Because $t > 0$

b) $p(t) = \int v(t)$

$= \int 4t^3 - 4t \, dt$

$= t^4 - 2t^2 + c$

22 cont.

$p(t) = t^4 - 2t^2 + c$

$p(1) = 3$ so

$3 = 1^4 - 2(1)^2 + c$

$3 = 1 - 2 + c$

$3 = -1 + c$

$4 = c$

$p(t)$ or $\boxed{x(t) = t^4 - 2t^2 + 4}$

c) $\int |v(t)|\, dt$ = Total distance Travelled

$\int_0^2 |4t^3 - 4t|\, dt$

$= \boxed{10 \text{ units travelled}}$

23. $\frac{ds}{dt} = 10$

$A = 150 \qquad \frac{dA}{dt} = ?$

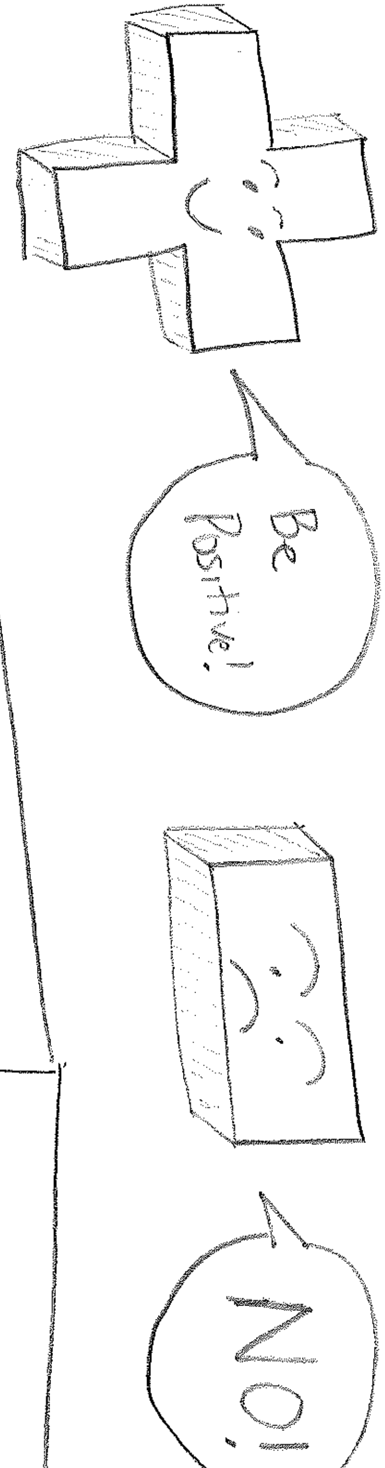
$150 = s^2$

$s = \sqrt{150}$

$A = s^2$

$\frac{dA}{dt} = 2s\frac{ds}{dt} = 2\sqrt{150}\,(10)$

$= 20\sqrt{150} \; \boxed{= 244.949 \text{ cm}^2/\text{sec}}$



This is an inspirational Quote because I have extra space.